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EUROPEAN ATOMIC ENERGY COMMUNITY - EURATOM

ON THE PARAMETRIC REPRESENTATION OF FAST REACTOR SPECTRA

by

G. BLÄSSER and E. DIANA

1964

Joint Nuclear Research Centre
Ispra Establishment - Italy
Reactor Physics Department

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On the parametric representation of fast reactor spectra

(*Received* 5 *October* 1962)

1. INTRODUCTION

UP TO the present time, fast reactor calculations have been based on the multigroup approach, starting from basic physical data such as cross sections as functions of energy. For thermal reactors, this method is often replaced by a few-group theory based on integral data. Besides reducing the amount of computer time, the integral approach uses information which usually is more easily obtained by the experiments. In the field of fast reactors many cross-sectional data are still unknown, or at least not known with the required precision. Yet, the integral method was seldom used for fast systems, since it was believed that results obtained in this way for one system do not apply to different systems because of the changes in the spectrum. However, if it is possible to represent the spectrum by a small number of parameters, one can use an integral approach for each parameter configuration and extend thereby the information gained from integral experiments to a large extent.

2. METHOD OF ANALYSIS

We calculated the spectrum Φ in the centre of a certain number of unreflected spherical uranium fuelled fast assemblies by a multigroup code (ZOOM-7090 version) using 16 groups and the cross sections as given by YIFTAH, OKRENT and MOLDAUER (1960). We assumed, heuristically, that the spectrum Φ can be written as a superposition of the normalized fission spectrum χ and a residual normalized spectrum ρ:

$$\Phi = A(\alpha\chi + \rho)$$

where A is a factor related to the normalization of Φ or the total flux level and α gives the ratio of the numbers of neutrons distributed according to the two different spectra χ and ρ respectively.

The residual spectrum ρ might be characterized in terms of its mean lethargy and its higher moments:

$$\bar{u} = \sum_i u_i \rho_i,$$

$$\mu_n = \sum_i (u_i - \bar{u})^n \rho_i$$

where u_i is the value of lethargy attributed to the group i.

In statistical literature, one often finds the following problem: given the moments up to the fourth, find a distribution curve with these moments. The simplest approach is that using the K. Pearson system of frequency curves (PEARSON and HARTLEY, 1958). There one classifies the frequency curves in eight types according to the values of β_1 and β_2 where:

$$\beta_1 = \mu_3^2/\mu_2^3,$$
$$\beta_2 = \mu_4/\mu_2^2.$$

3. RESULTS OF THE CALCULATIONS

We calculated the items mentioned in Section 2 for the following assemblies:

(a) Pure uranium metal assemblies with an enrichment varying between 94 and 10%.
(b) Uranium metal assemblies with an enrichment of 50%, containing sodium up to 50% in volume.
(c) One assembly containing 50% (vol.) Na, 20% steel and 30% uranium metal (50% enriched).
(d) Uranium oxide assemblies with an enrichment between 75% and 25%.
(e) One uranium oxide (50% enriched) assembly with 50% (vol.) Na.
(f) One uranium oxide (50% enriched) assembly with 50% (vol.) of Na and 20% (vol.) of steel.
(g) Uranium carbide assemblies with an enrichment between 75% and 25%.
(h) One uranium carbide (50% enriched) assembly with 50% (vol.) Na.
(i) One uranium carbide (50% enriched) assembly with 50% (vol.) of Na and 20% (vol.) of steel.

TABLE 1.—VALUES OF α, $\bar{u}$, μ_2, μ_3, μ_4, CALCULATED FOR A NUMBER OF FAST ASSEMBLIES

Case	α	$\bar{u}$	μ_2	μ_3	μ_4
(a) 94% enr.	1·327	2·921	0·771	0·152	1·667
75% enr.	1·043	2·917	0·741	0·194	1·575
50% enr.	0·701	2·937	0·707	0·249	1·532
25% enr.	0·375	3·029	0·708	0·332	1·668
10% enr.	0·165	3·251	0·816	0·483	2·243
(b) 30% vol. Na	0·759	2·972	0·758	0·211	1·772
50% vol. Na	0·617	2·978	0·757	0·251	1·780
(c)	0·406	3·008	0·880	0·341	2·465
(d) 75% enr.	0·561	2·965	1·033	0·274	2·990
50% enr.	0·416	3·123	1·087	0·369	3·508
25% enr.	0·262	3·443	1·301	0·634	5·156
(e)	0·382	3·222	1·186	0·441	4·269
(f)	0·249	3·354	1·371	0·772	6·005
(g) 75% enr.	0·830	3·014	0·955	0·319	2·634
50% enr.	0·576	3·107	0·997	0·456	3·053
25% enr.	0·331	3·321	1·190	0·767	4·521
(h)	0·504	3·172	1·081	0·502	3·643
(i)	0·309	3·243	1·255	0·744	5·111

The most striking result has been the relative linearity in the dependence of the parameter α on the enrichment down to enrichments of about 15–20 per cent (Fig. 1) and the smallness of the variation of the mean lethargy and the second moment (Table 1) with the change in core composition. It turned out, furthermore, that all spectra could be fitted by Type I curves of the Pearson system. To give an idea of the 'goodness of fit' a comparison between the exact reduced spectrum and such a representation by a Pearson Type I curve is given in Fig. 2 for a typical example.

The expression for a Type I frequency curve is:

$$\rho = \text{const.}\left(1 + \frac{x}{a_1}\right)^{m_1}\left(1 - \frac{x}{a_2}\right)^{m_2}$$

where the relation between the parameters m_1, m_2, a_1, a_2 and the moments is given in the Appendix and where x is the difference between the lethargy value u and the mode value, i.e. the value of lethargy for which the distribution attains its maximum.

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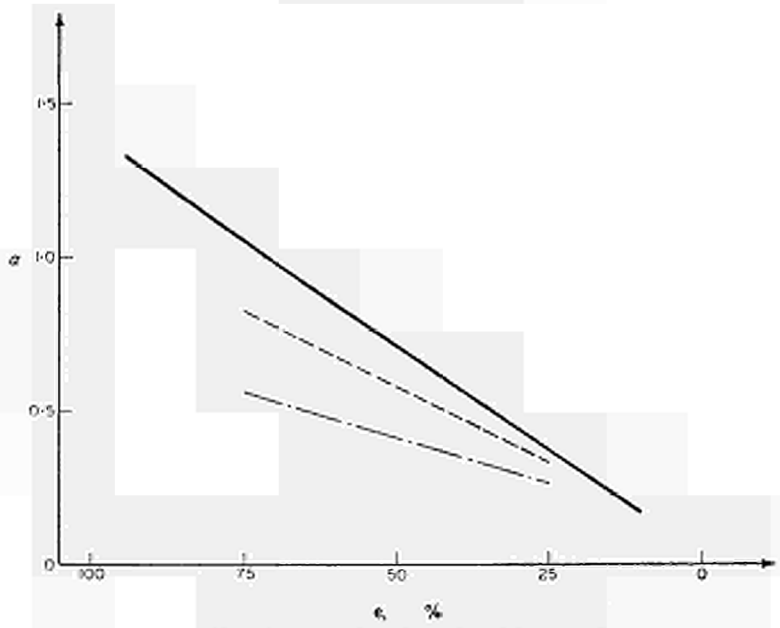

FIG. 1.—As a function of enrichment of:
——— uranium metal (case A)
— — — uranium carbide (case B)
— · — · — uranium oxide (case C).

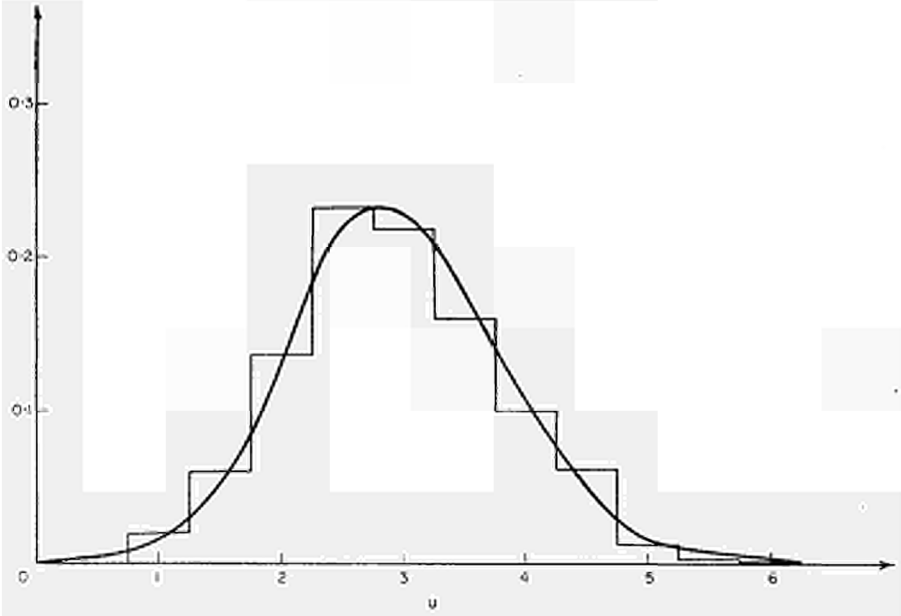

FIG. 2.—Comparison between the calculated reduced spectrum and the fitted frequency curve (case A 94% enriched).

APPENDIX

According to VIANELLI (1959), the position of the mode and the parameters of the Type I distribution curve are expressed in terms of the moments by:

$$\text{Mode} = a - \frac{1}{2}\cdot\frac{\mu_3}{\mu_2}\cdot\frac{r+2}{r-2},$$

where

$$r = \frac{6(\beta_2 - \beta_1 - 1)}{6 + 3\beta_1 - 2\beta_2}, \quad \beta_1 = \mu_3^2/\mu_2^3, \quad \beta_2 = \mu_4/\mu_2^2,$$

$$m_1 = \frac{1}{2}\left\{r - 2 - r(r+2)\sqrt{\frac{\beta_1}{\beta_1(r+2)^2 + 16(r+1)}}\right\},$$

$$m_2 = \frac{1}{2}\left\{r - 2 + r(r+2)\sqrt{\frac{\beta_1}{\beta_1(r+2)^2 + 16(r+1)}}\right\},$$

$$a_1 = \frac{m_1 b}{m_1 + m_2} = \frac{m_1 b}{r-2},$$

$$a_2 = \frac{m_2 b}{m_1 + m_2} = \frac{m_2 b}{r-2},$$

where

$$b = a_1 + a_2 = \tfrac{1}{2}\sqrt{\mu_2}\cdot\sqrt{\beta_1(r+2)^2 + 16(r+1)}.$$